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Possible Determination of the η^0 Lifetime with Electron-Positron Colliding Beams.

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1. – The experimental determination of the η^0 lifetime appears to be a rather hard problem. The accepted set of quantum numbers for $\eta^0(J=0,\ P=-1,\ I=0,\ C=G=+1)$ implies that its decays occur electromagnetically. A theoretical prediction (1) on the rate of $\eta^0\to 2\gamma$ (one of the dominant modes) is based on unitary symmetry and leads to a rate $w(\eta^0\to 2\gamma)\simeq 22w(\pi^0\to 2\gamma)$. For a π^0 lifetime $\sim 1.3\cdot 10^{-16}$ s, and with the assumption that $\eta^0\to 2\gamma$ occurs about 40% of the times, one finds a lifetime for η^0 of $2.4\cdot 10^{-18}$ s. The experimental upper limit on the width only excludes lifetimes shorter than 10^{-22} s (2).

A possible method for determining the η^0 lifetime has been suggested by BELLETTINI *et al.* (3) and it is based on the coherent η^0 production by a photon on the Coulomb field of a nucleus (Primakoff effect). The method may actually work provided one disposes of a photon beam of relatively high intensity and of energy of $(4\div 5)$ GeV, and one can separate the η^0 Primakoff effect on a heavy nucleus from all nuclear production events. It has also been suggested by CONTOGOURIS and VERGANELAKIS (4) that proton Compton scattering with polarized photons can give at least a lower limit on the η^0 lifetime. Perhaps more promising are possible experiments on the peripheral production of the $\pi+\eta$ system (5,6).

In this note we show that experiments with colliding electron positron beams, now in preparation in different laboratories, may allow a determination of the η^0 lifetime. We shall discuss the two reactions:

$$e^+ + e^- \to \eta^0 + \gamma\,, \tag{1}$$

$$e^+ + e^- \to \eta^0 + \pi^+ + \pi^-\,. \tag{2}$$

(1) N. CABIBBO and R. GATTO: *Nuovo Cimento*, **21**, 872 (1961).
(2) P. L. BASTIEN, *et al.*: *Phys. Rev. Lett.*, **8**, 114 (1962).
(3) G. BELLETTINI, C. BEMPORAD, P. L. BRACCINI, L. FOÀ and M. TOLLER: *Phys. Lett.*, **3**, 170 (1963).
(4) A. CONTOGOURIS and A. VERGANELAKIS: *Phys. Lett.*, **6**, 103 (1963).
(5) G. PUPPI: *Ann. Rev. Nuclear Sci.*, (1963) (edited by E. SEGRÈ, Palo Alto, California), p. 325.
(6) E. CELEGHINI and R. GATTO: *Phys. Lett.*, (to be published).

Both reactions are thought to occur through single photon exchange. From the first reaction one can measure the $\eta^0 \to 2\gamma$ vertex with one photon off-mass shell with a squared four-momentum $K^2 = -4E^2$, where E is the colliding beam energy ($2E$ is the total center-of-mass energy). From the second reaction one can measure the vertex of $\eta^0 \to \gamma + \pi^+ + \pi^-$, again with the photon off-mass shell and with $K^2 = -4E^2$. To relate the off-mass shell behaviour of the $\eta^0 \to 2\gamma$ and $\eta^0 \to \gamma + \pi^+ + \pi^-$ vertices to their values for real photons, which determine the partial lifetimes for these decay modes, we make reasonable guesses on the momentum dependence of the relevant form factors. Specifically we assume that the form factors are essentially determined by the resonant intermediate vector states (ρ and ω). On these assumptions reaction (2) appears to be particularly convenient, mainly because of the broad enhancement that occurs when the two final pions are produced in the resonant p-wave.

2. – We discuss first the reaction (1)

$$e^+ + e^- \to \eta^0 + \gamma .$$

We write the vertex $\eta^0 \to 2\gamma$ in its general form as

$$\tfrac{1}{4}\, g(K^2, q^2, p^2)\, \varepsilon_{\mu\nu\lambda\varrho} F_{\mu\nu}(q) F_{\lambda\varrho}(K) , \tag{3}$$

where K and q are the photon momenta and p is the η^0 momentum; $F_{\mu\nu}(q)$ is the appropriate Fourier component of the electromagnetic tensor; $\varepsilon_{\mu\nu\lambda\varrho}$ is the antisymmetric tensor; $g(K^2, q^2, p^2)$ is a form factor. We call $g(K^2)$ the form factor when one photon and the η^0 are on the mass-shell: $g(K^2) = g(K^2, 0, -m_\eta^2)$. For varying K^2, the form factor $g(K^2)$ appears in the description of different experiments. For $K^2 = 0$ all particles are on the mass-shell and the value of $|g(0)|$ is directly related to the $\eta^0 \to 2\gamma$ decay rate

$$w(\eta^0 \to 2\gamma) = \frac{m_\eta^3}{64\pi}\, |g(0)|^2 . \tag{4}$$

For $K^2 \geqslant 0$ the form factor is in principle obtainable from coherent η^0 production by a photon in the Coulomb field of a nucleus [7,3] (such as $\gamma + \mathrm{Pb} \to \eta^0 + \mathrm{Pb}$ occuring coherently through $\gamma + (\text{Coulomb field}) \to \eta^0$). In practice only very low values of K^2 ($K^2 \simeq 0$) will be reached in this experiment. For $-m_\eta^2 \leqslant K^2 \leqslant 0$ the values of $|g(K^2)|$ can in principle be obtained from the distributions of the invariant masses of $e^+ + e^-$ and of $\mu^+ + \mu^-$ in $\eta^0 \to \gamma + e^+ + e^-$ and $\eta^0 \to \gamma + \mu^+ + \mu^-$ [8]. For $K^2 \leqslant -m_\eta^2$ the value of $|g(K^2)|$ can be obtained from the colliding beam reaction $e^+ + e^- \to \eta^0 + \gamma$ at a total c.m. energy $2E = \sqrt{-K^2}$.

We assume that the resonant p-wave pion term dominates the absorptive part of $g(K^2)$ in a nonsubtracted dispersion relation. The form factor is then given by

$$g(K^2) = g(0)\, \frac{m_\varrho^2}{m_\varrho^2 + K^2 - i\gamma(-\frac{1}{4}K^2 - m_\pi^2)^{\frac{3}{2}}} \tag{5}$$

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(7) A. PRIMAKOFF: *Phys. Rev.*, **81**, 899 (1951).

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with $\gamma \simeq 0.4 m_\pi^{-1}$. The cross-section for $e^+ + e^- \to \eta^0 + \gamma$ in c.m. can be written as

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi\alpha}{m_\eta^3} w(\eta^0 \to 2\gamma) \beta_\eta^3 (1+\cos^2\theta) \left| \frac{g(-K^2)}{g(0)} \right|^2 , \tag{6}$$

where β_η is the c.m. η^0 velocity and θ is the production angle. The cross-section is proportional to the rate for $\eta^0 \to 2\gamma$. A measurement of $e^+ + e^- \to \eta + \gamma$ will principally be useful to provide an upper limit for such a rate.

One can make use of the relation

$$w(\eta \to 2\gamma) = \frac{1}{3}\left(\frac{m_\eta}{m_\pi}\right)^3 w(\pi^0 \to 2\gamma) \tag{7}$$

obtained from unitary symmetry [1] after correction for the phase-space. With a lifetime of $1.3 \cdot 10^{-16}$ s for $\pi^0 \to 2\gamma$ one obtains in this way a rate of $1.7 \cdot 10^{17}\ s^{-1}$ for $\eta \to 2\gamma$. In Fig. 1 we have reported (full line) the total cross-section for $e^+ + e^- \to \eta + \gamma$ for a form factor $g(K^2)$ as given by (5) normalized such that the $\eta^0 \to 2\gamma$ rate agrees with the above unitary-symmetry prediction. The cross-section expected on the basis of the unitary-symmetry prediction for the $\eta^0 \to 2\gamma$ rate is very small and would be hardly detectable. Conceivably, with the present design parameters of Adone, the process may become detectable if the rate for $\eta^0 \to 2\gamma$ is of the order of $\sim 10^{19}\ s^{-1}$, rather larger than the rate suggested from unitary symmetry. In any case the experiment will provide an upper limit to the η^0 rate.

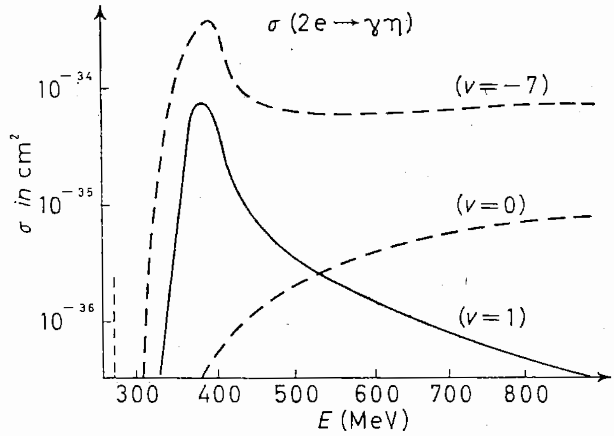

Fig. 1. – Full line: total cross section for $e^+ + e^- \to \eta^0 + \gamma$ with the form factor (5) for a rate of $1.7 \cdot 10^{17}\ s^{-1}$ $\eta^0 \to 2\gamma$. Dotted lines: total cross-section for $e^+ + e^- \to \eta^0 + \gamma$ with the form factor (8) for a rate of $1.7 \cdot 10^{17}\ s^{-1}$ for $\eta^0 \to 2\gamma$. For a different value of $\eta^0 \to 2\gamma$ rate the vertical scale must be changed proportionally.

In Fig. 1 the dotted lines give the total cross-sections for $e^+ + e^- \to \eta^0 + \gamma$ for a form factor $g(K^2)$ given by

$$g(K^2) = g(0)\left[v \frac{m_\rho^2}{m_\rho^2 + K^2 - i\gamma(-\frac{1}{4}K^2 - m_\pi^2)^{\frac{3}{2}}} + (1-v)\right] , \tag{8}$$

which contains also a subtraction term. For $v=1$ one has again the form factor with no subtraction term. The value $v=0$ corresponds to a constant form factor. A very tentative, and perhaps unreliable, determination of v was considered in ref. [8] by using SU_3 arguments to obtain the value of v from the measured derivative at the origin of the π^0 form factor with respect to the mass of the off-shell photon [9]. The value obtained in this way was $v = -(7 \pm 5)$. In ref. [8] the braching ratios ρ_e and ρ_μ for $\eta^0 \to \gamma + e^+ + e^-$ and $\eta^0 \to \gamma + \mu^+ + \mu^-$, respectively, relative to $\eta^0 \to 2\gamma$ were

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(8) E. Celeghini and R. Gatto: *Nuovo Cimento*, **28**, 1497 (1963).
(9) N. P. Samios: *Phys. Rev.*, **121**, 275 (1961).

determined as functions of v:

$$\varrho_\mu = (55.8 + 21.9v + 2.74v^2)\cdot 10^{-5} \qquad \text{and} \qquad \rho_e = (16.2 + 0.47v + 0.035v^2)\cdot 10^{-3} .$$

Measurements of these rates would determine v.

3. – Before discussing reaction (2) we shall examine here the decay mode

$$\eta^0 \to \pi^+ + \pi^- + \gamma ; \tag{9}$$

we call $p^{(0)} = (\boldsymbol{p}^{(0)}, iE_0)$ the η^0 momentum, $p^{(+)} = (\boldsymbol{p}^{(+)}, iE_+)$ and $p^{(-)} = (\boldsymbol{p}^{(-)}, iE_-)$ the π^+ and π^- momenta, and K, ε_μ the photon momentum and polarization vector. We write the decay matrix element as

$$\frac{1}{(2\pi)^2}\,\frac{\delta^4(p^{(0)} - p^{(-)} - p^{(+)} - K)}{\sqrt{16E_0E_+E_-}}\, G(s^2, t^2)\,\varepsilon_{\mu\nu\lambda\sigma}\,\varepsilon_\mu p_\nu^{(+)} p_\lambda^{(-)} p_\sigma^{(0)} , \tag{10}$$

where $G(s^2, t^2)$ is a form factor depending on $s^2 = -(p^{(+)} + p^{(-)})^2$ and $t^2 = -(p_0 + p_-)^2$. It is easy to see that only the s-channel can feel vector meson (ρ, ω) resonances: specifically an intermediate ρ-meson state can couple to $\eta^0 + \gamma$ and to $\pi^+ + \pi^-$. In the t-channel an intermediate ω state cannot couple to $\eta + \pi$ because of isotopic spin, and an intermediate ρ cannot couple to $\eta + \pi$ because of G-conservation. From these considerations we approximate the form factor $G(s^2, t^2)$ as

$$G(s^2, t^2) = \frac{f}{s^2 - m_\rho^2 + i\Gamma} , \tag{11}$$

where Γ is given, as in (5), by $\Gamma = \gamma(\frac{1}{4}s^2 - m_\pi^2)^{\frac{3}{2}}$. The spectrum in s^2, the square of the invariant mass of the $\pi^+ + \pi^-$ system, is reported in Fig. 2, in absolute units, with f^2 undetermined (left-hand vertical scale). The value of f^2 can be determined from a knowledge of the $(\eta^0 \to \pi^+ + \pi^- + \gamma)/(\eta^0 \to 2\gamma)$ braching ratio if the $\eta^0 \to 2\gamma$ rate is known. We take for the $(\eta^0 \to \pi^+ + \pi^- + \gamma)/(\eta^0 \to 2\gamma)$ ratio a value of 0.16 ([5]). For the $\eta^0 \to 2\gamma$ rate we use tentatively the value obtained from unitary symmetry (eq. (7)). The total rate for $\eta^0 \to \pi^+ + \pi^- + \gamma$, obtained by integration of the spectrum of Fig. 2, is

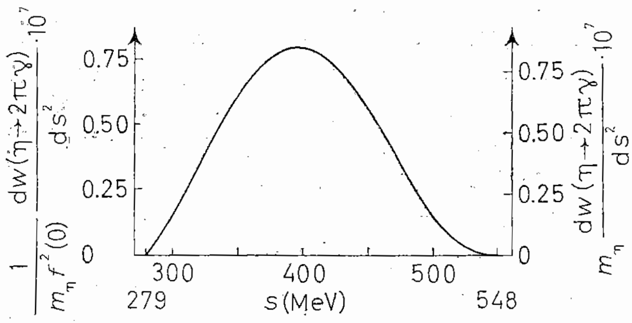

Fig. 2. – Invariant mass distribution of the π^+-π^- system in $\eta^0 \to \pi^+ + \pi^- + \gamma$. The right-hand vertical scale is obtained for a value $|f|^2 = 1.42\cdot 10^{-27}$ cm², corresponding to a rate of $1.7\cdot 10^{17}$ s⁻¹ for $\eta^0 \to 2\gamma$ and a ratio of 0.16 of $\eta^0 \to \pi^+ + \pi^- + \gamma$ with respect to this mode.

$$w(\eta^0 \to \pi^+\pi^-\gamma) = |f|^2 m_\eta^3 \cdot 3.03\cdot 10^{-8} . \tag{12}$$

Comparing with the rate predicted from the above branching ratio and from the unitary symmetry prediction for $\eta^0 \to 2\gamma$ we find $|f|^2 = 1.42\cdot 10^{-27}$ cm². The right-hand vertical scale in Fig. 2 is obtained for such a value of $|f|^2$.

4. – We now discuss the colliding beam reaction (2)

$$e^+ + e^- \to \pi^+ + \pi^- + \eta^0 .$$

We write the matrix element for reaction (2) at lowest electromagnetic order, as

(13) $$\frac{e}{(2\pi)^{\frac{7}{2}}} \frac{m_e \delta(q^{(+)} + q^{(-)} - p^{(0)} - p^{(+)} - p^{(-)})}{\sqrt{8E_0 E_+ E_- e_+ e_-}} G(s^2, t^2, K^2)\, \varepsilon_{\mu\nu\lambda\sigma} p_\nu^{(+)} p_\lambda^{(-)} p_\sigma^{(0)} \frac{1}{K^2} (\bar{u}\gamma_\mu v),$$

where $p^{(\pm)} = (\boldsymbol{p}^{(\pm)}, iE_\pm)$ are the pion momenta, $p^{(0)} = (\boldsymbol{p}^{(0)}, iE_0)$ is the η^0 momentum, $q^{(\pm)} = (\boldsymbol{q}^{(\pm)}, ie_+)$ are the electron and positron momenta, m_e is the electron mass, u and v are the electron and positron spinors, and K is the virtual photon momentum. The form-factor $G(s^2, t^2, K^2)$, with $s^2 = -(p^{(+)} + p^{(-)})^2$, $t^2 = -(p^{(0)} + p^{(-)})^2$ coincides with $G(s^2, t^2)$ when $K^2 = 0$

(14) $$G(s^2, t^2, 0) = G(s^2, t^2) .$$

The reaction (2) would permit a measurement of $|G(s^2, t^2, K^2)|^2$ for K^2 negative and less than $-(2m_\pi + m_\eta)^2$. For negative K^2 larger than $-(2m_\pi - m_\eta)^2$ the form factor $G(s^2, t^2, K^2)$ appears in the description of $\eta^0 \to \pi^+ + \pi^- + e^+ + e^-$ (8). This process however occurs as an internal conversion of the γ of $\eta^0 \to \pi^+ + \pi^- + \gamma$, and thus only very low values of K^2 are important. For $K^2 = 0$ the form factor G appears in the description of $\eta^0 \to \pi^+ + \pi^- + \gamma$, that we have already discussed. It also appears in the peripheral contribution to $\gamma + p \to \pi^+ + n + \eta^0$ through single pion exchange (6). The determination of the form factor for $K^2 > 0$, for instance through $\pi^+ \to \eta + \pi^+$ in the Coulomb field of a nucleus appears particularly difficult.

From G invariance one sees that the K^2 dependence in the form factor $G(s^2, t^2, K^2)$ may be strongly affected by the ρ-meson resonance (but not from the ω). Taking into account the considerations developed in the preceding section for $G(s^2, t^2)$, which led us to the form (11) for such a form factor, we assume for $G(s^2, t^2, K^2)$ the form

(15) $$G(s^2, t^2, K^2) = \frac{h}{(s^2 - m_\rho^2 + i\Gamma)(K^2 - m_\rho^2 + i\Gamma)} ,$$

where h is related to f by

(16) $$f = \frac{h}{-m_\rho^2 + i\Gamma} .$$

With our determination of $|f|^2$, obtained from the unitary symmetry prediction of $\eta^0 \to 2\gamma$ and from a ratio of 0.16 for $(\eta^0 \to 2\pi + \gamma)/(\eta^0 \to 2\gamma)$, we find $|h|^2 = 59 m_\pi^2$.

In Fig. 3. we give the K^2 spectrum for $\eta^0 \to \pi^+ + \pi^- + e^+ + e^-$ (K^2 is the invariant mass of $e^+ + e^-$) calculated

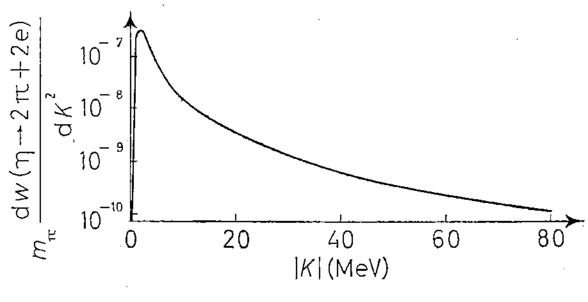

Fig. 3. – Distribution of the invariant $e^+ + e^-$ mass in $\eta^0 \to \pi^+ + \pi^- + e^+ + e^-$, assuming a rate of $1.7 \cdot 10^{17}\, s^{-1}$ for $\eta^0 \to 2\gamma$ and a value of 0.16 for $(\eta^0 \to 2\pi + \gamma)/(\eta^0 \to 2\gamma)$.

from the matrix element (13) with the form factor (15), with the above value of $|h|^2 = 59\, m_\pi^2$.

The cross-section of (2) in the c.m. system is given by

$$\text{(17)} \qquad \frac{d\sigma}{dE_+\, dE_-\, d(\cos\theta)} = \frac{\alpha}{(2\pi)^2}\, \frac{G(s^2, t^2, K^2)}{256}\, (\boldsymbol{p}^{(+)} \wedge \boldsymbol{p}^{(-)})^2 \sin^2\theta\,,$$

where θ is the angle between the incident electron momentum and the normal to the production plane. The angle φ between $\boldsymbol{p}^{(+)}$ and $\boldsymbol{p}^{(-)}$ is given by

$$\text{(18)} \qquad \cos\varphi = \frac{1}{2\,|\boldsymbol{p}^{(+)}|\,|\boldsymbol{p}^{(-)}|}\,[4E^2 - 4E(E_+ + E_-) + 2E_+E_- - m_\eta^2 + 2m_\pi^2]\,,$$

where $E = e_+ = e_-$ is the colliding beam energy. In (17) the virtual photon momentum K^2 is given by

$$K^2 = -4E^2\,.$$

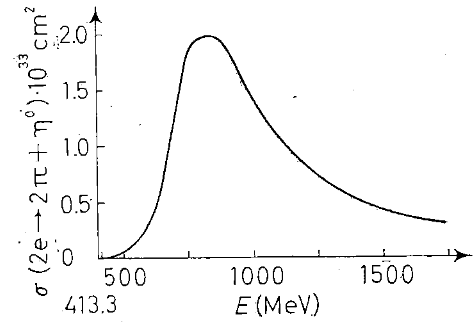

Fig. 4. – The total cross-section for $e^+ + e^- \to \pi^+ + \pi^- + \eta^0$ as a function of the colliding beam energy E. The cross-section has been evaluated assuming a rate of $w = 1.7 \cdot 10^{17}\ \text{s}^{-1}$ for $\eta^0 \to 2\gamma$ and a value of 0.16 for $R = (\eta^0 \to 2\pi\gamma)/(\eta^0 \to 2\gamma)$. The cross-sections for other values of w and R are simply obtained by altering the vertical scale proportionally to the product wR.

The total cross-section has been calculated with the expression (15) for the form factor $G(s^2, t^2, K^2)$. The curve reported in Fig. 4 has been obtained assuming the value $|h|^2 = 59 m_\pi^2$, corresponding to a rate of $1.7 \cdot 10^{17}\ \text{s}^{-1}$ for $w(\eta^0 \to 2\gamma)$ and a ratio R of 0.16 for $\eta^0 \to 2\pi + \gamma$ to $\eta^0 \to 2\gamma$. According to (16) and (12) the cross-section is proportional to both $w(\eta^0 \to 2\gamma)$ and R. Thus one can obtain curves corresponding to other choices of $w(\eta^0 \to 2\gamma)$ and R by multiplying by a factor the curve given in Fig. 4. The broad maximum in Fig. 4 can simply be explained through the occurrence of the enhanced channel $e^+ + e^- \to \rho^0 + \eta^0$, whose threshold lies at

$$E = \tfrac{1}{2}(m_\rho + m_\eta) \simeq 650\ \text{MeV}\,.$$

The pole at $K^2 = -m_\rho^2$ lies below threshold and only affects the initial rise of the cross-section.